Aplia - Ch. 8 #5

$Q = 4LK$ $\qquad$ $TC = rK + wL$

$MP_L = 4K$

$MP_K = 4L$

The firm is to minimize the cost of producing any output Q

Long-run cost minimization requires that we satisfy the tangency condition:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

$$\frac{4K}{w} = \frac{4L}{r}$$

$$\Downarrow$$

$$\boxed{K = \frac{w}{r} L}$$

Recall

$$K = \frac{w}{r} L \quad \text{(from previous page)}$$

Also

$$Q = 4LK \quad \text{(the production function)}$$

$$Q = 4L\left(\frac{w}{r}\right)L$$

$\Downarrow$ simplify, solve for L

$$L = \frac{1}{2}\sqrt{\frac{rQ}{w}} = \frac{1}{2} Q^{1/2}\left(\frac{r}{w}\right)^{1/2}$$

Again recall

$$K = \frac{w}{r} L$$

$$K = \frac{w}{r}\left(\frac{1}{2}\right) Q^{1/2}\left(\frac{r}{w}\right)^{1/2}$$

$$K = \left(\frac{r}{w}\right)^{-1}\left(\frac{1}{2}\right) Q^{1/2}\left(\frac{r}{w}\right)^{1/2}$$

$$K = \frac{1}{2} Q^{1/2}\left(\frac{r}{w}\right)^{-1/2}$$

$$K = \frac{1}{2}\sqrt{\frac{wQ}{r}}$$

We now have:

$$L = \frac{1}{2} Q^{1/2} \left(\frac{r}{w}\right)^{1/2}$$

$$K = \frac{1}{2} Q^{1/2} \left(\frac{r}{w}\right)^{-1/2}$$

Let $w = 8$ and $r = 18$

$$L = \frac{1}{2} Q^{1/2} \left(\frac{18}{8}\right)^{1/2}$$

$$L = \frac{1}{2} Q^{1/2} \left(\frac{\cancel{2} \cdot 9}{\cancel{2} \cdot 4}\right)^{1/2}$$

$$L = \frac{3}{2} \cdot \frac{1}{2} Q^{1/2}$$

$$\boxed{L = 3/4\, Q^{1/2}}$$

$$K = \frac{1}{2} Q^{\frac{1}{2}} \left(\frac{18}{8}\right)^{-1/2}$$

$$K = \frac{1}{2} Q^{1/2} \left(\frac{8}{18}\right)^{1/2}$$

$$K = \frac{1}{2} Q^{1/2} \left(2/3\right)$$

$$\boxed{K = 1/3\, Q^{1/2}}$$

$$TC = rK + wL$$

$$= 18 \cdot \frac{1}{3} \cdot Q^{1/2} + 8\left(\frac{3}{4}\right)Q^{1/2}$$

$$= 6Q^{1/2} + 6Q^{1/2}$$

$$TC = 12Q^{1/2}$$

$$AC = \frac{TC}{Q} = \frac{12Q^{1/2}}{Q}$$

$$AC = 12Q^{-1/2}$$