(1) $m_t - p_t = \gamma + c_2 R_t$

$$R_t = r + E_{t+1}(p_{t+1} - p_t) + \eta_t$$

$$m_t = \mu_0 + \mu_1 t + e_t$$

Substitute:

$$\mu_0 + \mu_1 t + e_t - p_t = \gamma + c_2\left[r + E_t(p_{t+1} - p_t) + \eta_t\right]$$

$$\mu_0 + \mu_1 t + e_t = \gamma + c_2\left[r + E_t(p_{t+1} - p_t) + \eta_t\right] + p_t$$

Conjecture:

$$p_t = \phi_0 + \phi_1 t + \phi_2 e_t + \phi_3 \eta_t$$

$$p_{t+1} = \phi_0 + \phi_1(t+1) + \phi_2 e_{t+1} + \phi_3 \eta_{t+1}$$

$$E_t p_{t+1} = \phi_0 + \phi_1(t+1)$$

$$E_t(p_{t+1} - p_t) = \phi_0 + \phi_1(t+1) - \phi_0 - \phi_1 t - \phi_2 e_t - \phi_3 \eta_t$$

$$= \phi_1 - \phi_2 e_t - \phi_3 \eta_t$$

Substitute.

$$\mu_0 + \mu_1 t + e_t = \gamma + c_2\left[r + \phi_1 - \phi_2 e_t - \phi_3 \eta_t + \eta_t\right] + \phi_0 + \phi_1 t + \phi_2 e_t + \phi_3 \eta_t$$

Equate coefficients:

① $\mu_0 = \gamma + c_2 r + c_2 \phi_1 + \phi_0$ (constant)

② $\mu_1 = \phi_1$ (t)

③ $1 = -c_2 \phi_2 + \phi_2$ (e_t)

④ $0 = -c_2 \phi_3 + c_2 + \phi_3$ (n_t)

Solve

④ $\Rightarrow$ $-c_2 \phi_3 + c_2 + \phi_3 = 0$

$(-c_2 + 1)\phi_3 = -c_2$

$$\boxed{\phi_3 = \frac{-c_2}{1 - c_2}}$$

③ $\Rightarrow$ $1 = -c_2 \phi_2 + \phi_2$

$1 = (1 - c_2)\phi_2$

$$\boxed{\phi_2 = \frac{1}{1 - c_2}}$$

② $\Rightarrow$ $\boxed{\phi_1 = \mu_1}$

① $\Rightarrow$ $\mu_0 = \gamma + c_2 r + c_2 \phi_1 + \phi_0$

$\mu_0 = \gamma + c_2 r + c_2 \mu_1 + \phi_0$

$$\boxed{\phi_0 = \mu_0 - (\gamma + c_2 r + c_2 \mu_1)}$$

Solution is:

$$\phi_0 = \mu_0 - (\gamma + c_2 r + c_2 \mu_1)$$

$$\phi_1 = \mu_1$$

$$\phi_2 = \frac{1}{1-c_2}$$

$$\phi_3 = \frac{-c_2}{1-c_2}$$

Constant

t

e_t

η_t

(2) $y_t = \beta_0 + \beta_1 (m_t - p_t) + v_t$

$y_t = \bar{y} + \alpha (p_t - E_{t-1} p_t)$

$m_t = \mu_0 + \mu_1 v_{t-1} + e_t$

Substitute:

$\beta_0 + \beta_1 (\mu_0 + \mu_1 v_{t-1} + e_t) + v_t = \bar{y} + (\alpha + \beta_1) p_t - \alpha E_{t-1} p_t$

Conjecture:

$p_t = \phi_0 + \phi_1 e_t + \phi_2 v_t + \phi_3 v_{t-1}$

$E_{t-1} p_t = \phi_0 + \phi_3 v_{t-1}$

Substitute

$\beta_0 + \beta_1 (\mu_0 + \mu_1 v_{t-1} + e_t) + v_t = \bar{y} + (\alpha + \beta_1)(\phi_0 + \phi_1 e_t + \phi_2 v_t + \phi_3 v_{t-1}) - \alpha (\phi_0 + \phi_3 v_{t-1})$

Equate coefficients:

① $\beta_0 + \beta_1 \mu_0 = \bar{y} + (\alpha + \beta_1)\phi_0 - \alpha\phi_0$ (constant)

② $\beta_1 = (\alpha + \beta_1)\phi_1$ (e_t)

③ $1 = (\alpha + \beta_1)\phi_2$ (v_t)

④ $\beta_1 \mu_1 = (\alpha + \beta_1)\phi_3 - \alpha\phi_3$ (v_{t-1})

Solve:

④ $\Rightarrow \beta_1 \mu_1 = (\alpha + \beta_1 - \alpha)\phi_3$

$\beta_1 \mu_1 = \beta_1 \phi_3$

$\boxed{\phi_3 = \mu_1}$ (v_{t-1})

③ $\Rightarrow 1 = (\alpha + \beta_1)\phi_2$

$\boxed{\phi_2 = \frac{1}{\alpha + \beta_1}}$ (v_t)

② $\Rightarrow \beta_1 = (\alpha + \beta_1)\phi_1$

$\boxed{\phi_1 = \frac{\beta_1}{\alpha + \beta_1}}$ (e_t)

① $\Rightarrow \beta_0 + \beta_1 \mu_0 = \bar{y} + (\alpha + \beta_1)\phi_0 - \alpha\phi_0$

$\beta_0 + \beta_1 \mu_0 = \bar{y} + \beta_1 \phi_0$

$\beta_0 + \beta_1 \mu_0 - \bar{y} = \beta_1 \phi_0$

$\boxed{\phi_0 = \frac{\beta_0 + \beta_1 \mu_0 - \bar{y}}{\beta_1}}$ (constant)

Solution is

$$\phi_0 = \frac{\beta_0 + \beta_1 M_0 - \bar{y}}{\beta_1}$$ constant

$$\phi_1 = \frac{\beta_1}{\alpha + \beta_1}$$ e_t

$$\phi_2 = \frac{1}{\alpha + \beta_1}$$ v_t

$$\phi_3 = \beta_1 M_1$$ v_{t-1}

Output solution

$$y = \bar{y} + \alpha \left(P_t - E_{t-1} P_t \right)$$

$$y = \bar{y} + \alpha \left(\phi_1 e_t + \phi_2 v_t \right)$$

Note: Output does <u>not</u> depend on v_{t-1}

Quick Answers

(1)

$$P_t = \phi_0 + \phi_1 t + \phi_2 e_t + \phi_3 n_t$$

ϕ_0	$\mu_0 - (\gamma + c_2 r + c_2 \mu_1)$	Constant
ϕ_1	μ_1	t
ϕ_2	$\frac{1}{1-c_2}$	e_t
ϕ_3	$\frac{-c_2}{1-c_2}$	n_t

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(2) $P_t = \phi_0 + \phi_1 e_t + \phi_2 v_t + \phi_3 v_{t-1}$

ϕ_0	$\frac{\beta_0 + \beta_1 \mu_0 - \bar{y}}{\beta_1}$	Constant
ϕ_1	$\frac{\beta_1}{\alpha + \beta_1}$	e_t
ϕ_2	$\frac{1}{\alpha + \beta_1}$	v_t
ϕ_3	μ_1	v_{t-1}