Lucas Empirical Study/Barro-Rush Empirical Study

Lucas

Suppliers are located in a large number of scattered, competitive markets.
Demand for goods in each period is distributed unevenly over markets, leading to relative as well as general price movements.
Quantity supplied in each market is the product of a normal (secular) component common to all markets and a cyclical component which varies from market to market. In logarithms:

$$y_t(z) = y_{nt} + y_{ct}(z). \qquad (1)$$

Throughout the analysis, z is an index referring to local markets.

The secular component of output follows the trend line:

$$y_{nt} = a + bt. \qquad (2)$$

Supply in market *z* depends on the perceived relative price of good *z*.

$$y_t(z) = y_{nt} + g\left[P_t(z) - E\left(P_t | I_t(z)\right)\right] + l\left[y_{t-1}(z) - y_{n,t-1}\right]. \qquad (3)$$

This equation says that supply in market *z* depends on the perceived relative price of good *z*. The local price, $P(z_t)$, is observed at time *t*, but the aggregate price level, P_t, is not. Thus, individuals in market *z* form an expectation about P_t based on the information available to them at time *t* in market *z*. Note that price variables are in logs also.

Substituting (2) into (3), we get (3)':

$$y_t(z) = a + bt + g\left[P_t(z) - E\left(P_t | I_t(z)\right)\right] + l\left[y_{t-1}(z) - a - b(t-1)\right] \qquad (3)'$$

Each market's price is subject to a real market-specific shock:

$$P_t(z) = P_t + z_t \qquad (4)$$

(Note that *z* is an index of markets, z_t is the market specific shock in market *z)*.

We will show that:

$$E\left(P_t | I_t(z)\right) = (1-q)P_t(z) + q\overline{P}_t, \qquad (5)$$

where:

$\overline{P}_t$ = the expected value of the general price level at time t , with the expectation based on knowledge common to all traders (but not based on any market specific price information),

$q = \dfrac{t^2}{s^2 + t^2}$;

s^2 = the variance of the general price level, P_t;

t^2 = the variance of the market specific shock, z_t.

The result above is just a consequence of our analysis of the signal extraction problem.

$$P_t(z) = P_t + z_t$$
$$P_t(z) - \overline{P}_t = \left(P_t - \overline{P}_t\right) + z_t$$

(Now all variables are expressed as deviations from means)

Our forecast of $P_t - \overline{P}_t$ is $\left(\dfrac{s^2}{s^2 + t^2}\right)\left(P_t(z) - \overline{P}_t\right)$ so

$$E\left(P_t | I_t(z)\right) = \overline{P}_t + \left(\frac{s^2}{s^2 + t^2}\right)\left(P_t(z) - \overline{P}_t\right)$$

$$= \overline{P}_t + (1-q)\left(P_t(z) - \overline{P}_t\right)$$
$$= \overline{P}_t + (1-q)P_t(z) - (1-q)\overline{P}_t$$
$$= (1-q)P_t(z) + q\overline{P}_t \,,$$

which is what we wished to show.

Substituting (4) into (3)':

$$y_t(z) = a + bt + g\left[P_t(z) - (1-q)P_t(z) - q\overline{P}_t\right] + l\left[y_{t-1}(z) - a - b(t-1)\right]$$
$$y_t(z) = a + bt + g\left[qP_t(z) - q\overline{P}_t\right] + l\left[y_{t-1}(z) - a - b(t-1)\right]$$
$$y_t(z) = a + bt + gq\left[P_t(z) - \overline{P}_t\right] + l\left[y_{t-1}(z) - a - b(t-1)\right]$$

Now average over markets to get the aggregate supply relation:

$$y_t = a + bt + gq\left[P_t - \overline{P}_t\right] + l\left[y_{t-1} - a - b(t-1)\right] \qquad (7)$$

Now assume an aggregate demand equation:

$$x_t = y_t + P_t = x_{t-1} + d + e_t \qquad (8)$$

or

$$y_t = x_{t-1} + d + e_t - P_t \qquad (8)'$$

Combine (7) and (8), and rearrange:

$$a + bt + gq\left[P_t - \overline{P}_t\right] + l\left[y_{t-1} - a - b(t-1)\right] = x_{t-1} + d + e_t - P_t$$

or

$$a - al + lb + bt - lbt + (1 + qg)P_t - qg\overline{P}_t + ly_{t-1} = x_{t-1} + d + e_t$$

Now note that $\overline{P}_t = E_{t-1}P_t$, and conjecture a solution:

$$P_t = f_0 + f_1 t + f_2 x_{t-1} + f_3 y_{t-1} + f_4 e_t$$

$$E_{t-1}P_t = f_0 + f_1 t + f_2 x_{t-1} + f_3 y_{t-1}.$$

Substituting the expressions above into (6)' we get:

$$a - al + lb + bt - lbt + (1 + qg)\left[f_0 + f_1 t + f_2 x_{t-1} + f_3 y_{t-1} + f_4 e_t\right]$$
$$-qg\left[f_0 + f_1 t + f_2 x_{t-1} + f_3 y_{t-1}\right] + ly_{t-1} = x_{t-1} + d + e_t$$

Now equate coefficients on each side and solve for the f_i values.

The solutions are:

$$f_0 = d - lb - a(1 - l)$$

$$f_1 = -b(1 - l)$$

$$f_2 = 1$$

$$f_3 = -l$$

$$f_4 = \frac{1}{1+qg}$$

Check Lucas's solution on the top of p. 136. (Show that it is identical).

To get a solution for output, rewrite (7):

$$y_t = a + bt + gq\left[P_t - \overline{P}_t\right] + l\left[y_{t-1} - a - b(t-1)\right] \qquad (7)$$

$$y_t = y_{nt} + gq\left[P_t - \overline{P}_t\right] + l\left[y_{t-1} - y_{n,t-1}\right]$$

$$y_t = y_{nt} + gq\left[f_4 e_t\right] + l\left[y_{t-1} - y_{n,t-1}\right]$$

$$y_t = y_{nt} + gq\left[f_4 e_t\right] + l\left[y_{t-1} - y_{n,t-1}\right]$$

Substituting $e_t = \Delta x_t - d$, the solution for f_4, and recalling that $y_{nt} = a + bt$, the later equation can be rearranged to yield the output solution given by Lucas on p. 136:

$$y_t = -\frac{qgd}{1+gq} + lb + \frac{qg}{1+gq}\Delta x_t + l y_{t-1} + (1-l)y_{nt}$$

Alternative forms for the solution are:

$$y_{ct} = -pd + p\Delta x_t + l y_{c,t-1}$$

$$\Delta P_t = -b + (1-p)\Delta x_t + p\Delta x_{t-1} - l\Delta y_{c,t-1}$$

where

$$p = \frac{qg}{1+qg}$$

and

$$y_t(z) = y_{nt} + y_{ct}(z).$$

Recall that we assumed that P_t had unconditional mean $\overline{P}_t$ and variance s^2. From the solution for P_t, we see that the variance s^2 must be given by:

$$s^2 = \left(\frac{1}{1+gq}\right)^2 s_x^2, \qquad (9)$$

where s_x^2 is the variance of e_t. (see equation (8)).

Now let $p = \dfrac{gq}{1+gq}$ (this is the slope of the output solution equation).
Using the definition of q:

$$p = \frac{t^2 g}{s^2 + t^2(1+g)}.$$

Under the assumption that values for t^2 and g are similar across countries, one would expect values of p to decline with the variance s_x^2.

For his empirical work, Lucas uses this equation for output (a form of the solution obtained above):

$$y_{ct} = -pd + p\Delta x_t + l y_{c,t-1}$$

y_{ct} is the cyclical component of real output ($y_{ct} = y_t - y_{nt}$).
x_t is nominal GNP.
(Variables are logs).

The Lucas Model predicts that p will be smaller when the variance of *x* is larger. To test this hypothesis, Lucas estimated the key equation using time series data for each of 18 countries. Comparisons of coefficients were then made across country estimations. I.e., is the Phillips Curve tradeoff parameter smaller in a country where nominal income growth is more variable?

Results: Compare US and Argentina

$$y_{ct} = -0.049 + 0.910\Delta x_t + 0.887 y_{c,t-1} \qquad \text{US}$$

$$y_{ct} = -0.006 + 0.011\Delta x_t + 0.126 y_{c,t-1} \qquad \text{Argentina}$$

Barro and Rush: Empirical Tests of Lucas Model Predictions

Barro and Rush first estimate an equation to explain Money growth:

$$\dot{M} = f(\dot{M}_{-1}, \ldots, \dot{M}_{-6}, FEDV, U_{-1}, U_{-2}, U_{-3})$$

Next calculate predicted values for M and residuals:

DM is actual money growth.

$D\hat{M}$ is expected money growth.

DMR is unexpected money growth.

Then regress:

$$\log y_t = g\left(t, MIL, \log G_t, \frac{G_t}{y_t}, DMR_0, \ldots, DMR_{-10} \right)$$

Barro finds that money surprises are dignificant in explaining output (but anticipated money growth are not when those are added to the model).

Similar equations are used to explainthe unemployment rate (instead of output).