Solow on the Decomposition of Growth

$Y = F(K, L)$ (CRS)

$Y = A(t)F(K, L)$

Differentiate with respect to time:

$$\frac{dY}{dt} = \frac{dA}{dt}F(K,L) + A\left[\frac{\partial F}{\partial K}\frac{dK}{dt} + \frac{\partial F}{\partial L}\frac{dL}{dt}\right]$$

Divide by Y:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + A\frac{\partial F}{\partial K}\frac{\dot{K}}{Y} + A\frac{\partial F}{\partial L}\frac{\dot{L}}{Y}$$

Define $w_k = \frac{\partial Y}{\partial K}\frac{K}{Y}$ and $w_L = \frac{\partial Y}{\partial L}\frac{L}{Y}$, where $w_L + w_K = 1$, and substitute into the equation above.

(1) $$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + w_K\frac{\dot{K}}{K} + w_L\frac{\dot{L}}{L}$$

Put variables in intensive form:

$\frac{Y}{L} = y$, $\frac{K}{L} = k$.

We now need to use the following result:

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

We will show that this is true:

$$\dot{y} = \frac{d\left(\frac{Y}{L}\right)}{dt} = \frac{L\frac{dY}{dt} - Y\frac{dL}{dt}}{L^2}$$

$$\dot{y} = \frac{1}{L}\frac{dY}{dt} - \frac{1}{L}\frac{Y}{L}\frac{dL}{dt}$$

$$\frac{\dot{y}}{y} = \frac{L}{Y}\frac{1}{L}\frac{dY}{dt} - \frac{L}{Y}\frac{1}{L}\frac{Y}{L}\frac{dL}{dt}$$

$$\frac{\dot{y}}{y} = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

Similarly:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L}$$

Using these results, we can rewrite (1) as

$$\frac{\dot{y}}{y} + \frac{\dot{L}}{L} = \frac{\dot{A}}{A} + w_K\left(\frac{\dot{k}}{k} + \frac{\dot{L}}{L}\right) + w_L\,\frac{\dot{L}}{L}$$

Recall that $w_L + w_K = 1$, so:

$$\frac{\dot{y}}{y} + \frac{\dot{L}}{L} = \frac{\dot{A}}{A} + w_K\left(\frac{\dot{k}}{k}\right) + \frac{\dot{L}}{L}$$

Finally,

$$\frac{\dot{y}}{y} = \frac{\dot{A}}{A} + w_K\,\frac{\dot{k}}{k}\,.$$

In this equation, $\frac{\dot{y}}{y}$, w_K, and $\frac{\dot{k}}{k}$ are measurable, so $\frac{\dot{A}}{A}$ can be inferred (and we can account for sources of growth. We use the preceding equation to decompose growth of per capita output into a component due to technical change and that due to growth of capital per worker.

Results

1909-29: Technical change accounts for 0.90 percentage points of per capita growth per year.

1930-1949: Technical change accounts for 2.25 percentage points of per capita growth per year.

Technical change accounts for 7/8 of per capita growth; increased capital per worker accounts for just 1/8.

Technical change appears to be highly variable from year to year.

Technical change is neutral (the assumption about the specification of technical change in the production function seems to be appropriate).

Question: Is the neutrality of technical change (as assumed here) consistent with the assumption of labor augmenting technical change (required to make Solow's growth theory fit the facts) ?